## Fusion Pore Conductance: Experimental Approaches and **Theoretical Algorithms**

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ABSTRACT Time-resolved admittance measurements provide the basis for studies showing that membrane fusion occurs through the formation and widening of an initially small pore, linking two previously separated aqueous compartments. Here we introduce modifications to this method that correct the cell-pipette (source) admittance for attenuation and phase shifts produced by electrophysiological equipment. Two new approaches for setting the right phase angle are discussed. The first uses the displacement of a patch-clamp amplifier C-slow potentiometer for the calculation of phase. This calculation is based on amplitudes of observed and expected (theoretical) changes in the source admittance. The second approach automates the original phase adjustment, the validity of which we prove analytically for certain conditions. The multiple sine wave approach is modified to allow the calculation of target cell membrane parameters and the conductance of the fusion pore. We also show how this technique can be extended for measurements of the resting potential of the first (voltage-clamped) membrane. We introduce an algorithm for calculation of fusion pore conductance despite a concurrent change in the resistance of the clamped membrane. The sensitivity of the capacitance restoration algorithm to phase shift errors is analyzed, and experimental data are used to demonstrate the results of this analysis. Finally, we show how the phase offset can be corrected "off-line" by restoring the shape of the capacitance increment.

GLOSSARY	
а	Real part of Q
$A_2$	Real part of $Y_2$
$A_2(\omega_{\rm k})$	Values of $A_2$ for two-sine-wave stimulus
	in MSW $(k = 1, 2)$
$A_{ m err}$	Real part of $(Y_2)_{out}$
$A_{\mathrm{in}}$	Real part of the signal at the input of
	PSD
$A_{ m out}$	Real part of $Y_{\text{out}}$
$A_{t}$	Real part of $Y_t$
$A_{ m tbf}$	Real part of $Y_{tbf}$
$A_{\rm tbf}(\boldsymbol{\omega}_1)$	Real part of $Y_{tbf}$ for the first sine wave
	stimulus in MSW
$A_{\rm tbf}(\omega_2)$	Real part of $Y_{\text{tbf}}$ for the second sine
	wave stimulus in MSW
$A_{\mathrm{tbf}}(\omega_{\mathrm{k}})$	Values of the real part of $Y_{\text{tbf}}$ for two-
	sine-wave stimulus in MSW ( $k = 1, 2$ )
b	Imaginary part of $Q$
$B_2$	Imaginary part of $Y_2$
$B_2(\omega_{ m k})$	Values of $B_2$ for two sine-wave-stimulus
-	in MSW $(k = 1, 2)$
$B_{\rm err}$	Imaginary part of $(Y_2)_{out}$
$B_{ m in}$	Imaginary part of the signal at the input
	of PSD
$B_{\rm out}$	Imaginary part of $Y_{\text{out}}$
$B_{\rm t}$	Imaginary part of $Y_t$
$B_{ m tbf}$	Imaginary part of $Y_{tbf}$

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- $B_{\rm tbf}(\omega_1)$ Imaginary part of  $Y_{tbf}$  for the first sine wave stimulus in MSW
- $B_{\rm tbf}(\omega_2)$ Imaginary part of  $Y_{\text{tbf}}$  for the second sine wave stimulus in MSW
- $B_{\rm thf}(\omega_{\rm k})$ Values of the imaginary part of  $Y_{thf}$  for two-sine-wave stimulus in MSW (k =
  - Real part of  $Y_1$  for the first sine wave  $c_1$ stimulus in MSW
  - Real part of  $Y_1$  for the second sine wave  $c_2$ stimulus in MSW
  - Real parts of  $Y_1(\omega_k)$  (k = 1, 2) $C_{\mathbf{k}}$
  - $C_{\mathrm{err}}$ Error in the calculated value of  $C_{\rm m2}$  due to the phase offset
  - $C_{m1}$ Capacitance of the first (initially clamped) membrane
  - $\Delta C_{\rm m1}$ Change in  $C_{m1}$ 
    - $C_{\rm m2}$ Capacitance of the second (initially nonclamped) membrane
    - $C_{\rm sl}$ C-slow potentiometer in the capacitance neutralization circuitry of a patch-clamp amplifier
  - $\Delta C_{\rm sl}$ Increment of C-slow potentiometer of a patch-clamp amplifier
    - $d_1$ Imaginary part of  $Y_1$  for the first sine wave stimulus in MSW
    - Imaginary part of  $Y_1$  for the second sine  $d_2$ wave stimulus in MSW
    - $d_{\mathbf{k}}$ Imaginary parts of  $Y_1(\omega_k)$  (k = 1, 2)
    - $E_{\rm r}$ Resting potential of the first (initially clamped) membrane
    - Frequency of the stimulating sine wave f
    - $B_2$  value for the first sine wave stimulus  $g_1$ in modified MSW
    - $B_2$  value for the second sine wave  $g_2$ stimulus in modified MSW

 $Y_3$ 

 $Y_{\rm G}$ 

 $Y_{\text{comp}}$ 

$g_{ m k}$	Same as $A_2(\omega_k)$ $(k = 1, 2)$
$G1_{m1}$	Changed value of $G_{m1}$
$G_{\rm a}$	Access (pipette) conductance
$G_{ m DC}$	Direct current conductance
$G_{ m err}$	Error in the calculated value of $G_p$ due
011	to the phase offset
$G_{\mathrm{m}1}$	Conductance of the first (initially
1111	clamped) membrane
$\Delta G_{ m m1}$	Increment of $G_{\rm m1}$
$G_{\mathrm{m2}}$	Conductance of the second (initially
○ m2	nonclamped) membrane
$G_{p}$	Conductance of the fusion pore
$G_{\rm sr}$	G-series potentiometer in the capacitance
O <sub>sr</sub>	neutralization circuitry of a patch-clamp
	amplifier
h	
$h_1$	$A_2$ value for the first sine wave stimulus
1.	in modified MSW
$h_2$	$A_2$ value for the second sine wave
1	stimulus in modified MSW
$h_{\mathrm{k}}$	Same as $B_2(\omega_k)$
1	Imaginary part of $(Y_{incr})_{out}$
$I_{ m h}$	Holding current
$I_{ m k}$	Imaginary parts of $[Y_{incr}(\omega_k)]_{out}$
J	Increment of $Y_t$ during the C-slow jump
$J_{ m out}$	J as measured from the output of PSD
$J1_{\mathrm{out}}$	$J_{\mathrm{out}}$ when phase deviates from $\varphi_{\mathrm{NM}}$
K	Factor that represents attenuation of $Y_t$
	caused by equipment
Q	Ratio of $J$ and $J_{\text{out}}$
R	Real part of $(Y_{incr})_{out}$
$R_{\rm a}$	Access (pipette) resistance
$R_{\rm k}$	Real parts of $[Y_{incr}(\omega_k)]_{out}$ $(k = 1, 2)$
$R_{\rm m1}$	Resistance of the first (initially clamped)
1111	membrane
$R_{\rm m2}$	Resistance of the second (initially
1112	nonclamped) membrane
$R_{\rm p}$	Fusion pore resistance
$T(\omega)^{p}$	Scaling factor as defined in Eq. 21
$T(\omega_{\rm k})$	Values of $T(\omega)$ for both sine wave
- ( k)	stimuli in modified MSW ( $k = 1, 2$ )
$U_{\mathrm{h}}$	Holding potential
$Y_1$	Admittance of $R_a$ and $Y_{m1}$ in series
$Y_1(\omega_k)$	Values of $Y_1$ for two-sine-wave stimulus
$I_1(\omega_k)$	in MSW $(k = 1, 2)$
V1	Admittance of the compensation
$Y1_{\text{comp}}$	
V1	circuitry after a C-slow jump is issued
$Y1_{t}$	Total admittance after a C-slow jump is
17	Admittages of C and V in society
$Y_2$	Admittance of $G_p$ and $Y_{m2}$ in series
$(Y_2)_{\text{out}}$	Value of $Y_2$ measured at the output of
T7 ( )	PSD
$Y_2(\omega_{\rm k})$	Values of $Y_2$ for two-sine-wave stimulus
	in MSW $(k=1,2)$

in MSW (k = 1, 2)

neutralization circuitry

Sum of  $Y_2$  and  $\Delta G_{m1}$ 

Admittance of  $R_a$  and  $Y_{m1+2}$  in series

Admittance of the capacitance

$Y_{\rm in}$	Signal at the input of the PSD
$Y_{\rm incr}$	Fusion-induced increment of the total
	admittance, $Y_t$
$(Y_{\rm incr})_{\rm out}$	$Y_{\rm incr}$ , as recorded from the output of
	PSD
$[Y_{\rm incr}(\omega_{\rm k})]_{\rm out}$	Values of $(Y_{incr})_{out}$ for both sine wave
	stimuli in modified MSW
$Y_{\rm m1}$	Admittance of the first (initially
	clamped) membrane
$Y_{m1+2}$	Admittance of $Y_{m1}$ and $Y_2$ in parallel
$Y_{\rm m2}$	Admittance of the second (initially
	nonclamped) membrane
$Y_{\rm out}$	Signal from the output of PSD
$\Delta Y_{ m out}(arphi)$	$\Delta C_{\rm m1}$ -induced change in admittance $Y_{\rm tbf}$
	as recorded from the output of PSD
$Y_{t}$	Total admittance of the general circuit
	shown in Fig. 1
$Y_{ m tbf}$	Admittance of $Y_{\text{comp}}$ and $Y_1$ in parallel;
	represents the total admittance $Y_t$ before
	fusion occurs
$\Delta Y_{ m tbf}$	Alteration in $Y_{\text{tbf}}$ induced by a change in
	$C_{\rm m1}$

### **Greek symbols**

ω	Angular frequency of the stimulating sine wave
$\omega_{ m k}$	Angular frequencies of both sine wave stimuli
	in MSW $(k = 1, 2)$
$\phi$	Argument of $T(\omega)$
$\varphi$	Phase angle introduced by equipment and PSD
$\omega_1$	Angular frequency of the first sine wave
	stimulus in MSW
$\varphi 1_{\mathrm{psd}}$	Any $\varphi_{\rm psd}$ not equal to $\varphi_{\rm NM}$
$\omega_2$	Angular frequency of the second sine wave
	stimulus in MSW
$arphi_{ m eq}$	Phase shift introduced by equipment (but not
•	PSD)
$arphi_{ m err}$	Error in the phase setting
$arphi_{ m NM}$	Phase set after NM approach was used
$arphi_{ m PL}$	Phase, satisfying requirements of PL technique

Phase shift introduced by a PSD

### INTRODUCTION

 $\varphi_{
m psd}$ 

Admittance analysis of single small cells, introduced in the early 1980s to study exocytosis (Neher and Marty, 1982), remains a leading technique in the field of biological membrane fusion. This method, called time-resolved admittance measurement (TRAM) (Gillis, 1995), involves stimulating the cell with a sine wave in voltage, separating the in-phase and out-of-phase components of the resultant current by a phase-sensitive detector (PSD), and calculating the electrical parameters of an equivalent model circuit. Later, this model was augmented by an additional element, the fusion pore conductance (Zimmerberg et al., 1987), which explains a correlated wave of the in-phase component and a slow rise in the out-of-phase component during each fusion event. More importantly, these admittance data led to the concept that membrane fusion occurs through the formation and widening of an initially narrow pore. Fusion pores were also measured by a direct current method, the "double whole-cell" recording (Lanzrein et al., 1993). Recently, comparison of the pore conductance measured by the direct current technique to that calculated using TRAM validated the model-dependent admittance analysis (Plonsky and Zimmerberg, 1996).

Since the invention of TRAM, different strategies for calculating the electrical parameters of fusing membranes have been suggested (see Lindau, 1991; Gillis, 1995, for reviews). The two most popular are the Lindau-Neher technique (LN) and the piecewise-linear technique (PL). In their initial form, both approaches neglect the fusion pore.

In LN,  $R_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$  are computed while a cell is stimulated with a single sine wave superimposed on a holding potential, assuming that  $E_{\rm r}$  is known and constant (symbols are cited in the legend to Fig. 1). A more general version of this technique uses a stimulus signal containing two sine waves with different frequencies, yielding calculation of  $R_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$  without any restrictions on  $E_{\rm r}$  (Rohlicek and Rohlicek, 1993; Donelly, 1994; Rohlicek and Schmid, 1994). Below, this generalized LN will be called

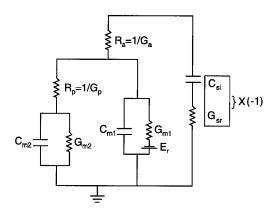


FIGURE 1 General equivalent circuit representing membrane fusion. Here  $R_a$  is the access (pipette) resistance,  $C_{m1}$  and  $R_{m1}$  are the capacitance and resistance of the first (patched and voltage-clamped) cell (or membrane),  $E_{\rm r}$  is its resting potential,  $C_{\rm m2}$  and  $R_{\rm m2}$  are the capacitance and the resistance of the second initially nonclamped membrane (cell or intracellular granule), and  $R_p$  is the fusion pore resistance.  $C_{sl}$  and  $G_{sr}$  represent the C-slow and G-series potentiometers in the capacitance neutralization (compensation) circuitry of a patch-clamp amplifier.  $C_{\rm sl}$  and  $G_{\rm sr}$  in series are assumed to have a "negative" admittance  $Y_{\rm comp}$ , which means that the addition of 1 pF in  $C_{\rm sl}$  value will be seen as subtraction of 1 pF at the output of an amplifier. We denote the total admittance of the general circuit above as  $Y_t$ , the admittance of the first membrane as  $Y_{m1}$  ( $Y_{m1}$  consists of  $C_{m1}$ ,  $G_{\mathrm{m1}}$ , and  $E_{\mathrm{r}}$ ), and the admittance of the second membrane as  $Y_{\mathrm{m2}}$  ( $Y_{\mathrm{m2}}$ includes  $C_{\rm m2}$  and  $G_{\rm m2}$ ).  $Y_{\rm m1}$  in series with  $R_{\rm a}$  forms  $Y_{\rm 1}$ , and  $Y_{\rm m2}$  in series with  $G_p$  forms  $Y_2$ .  $Y_{m1+2}$  stands for the resulting admittance of  $Y_{m1}$  and  $Y_2$ in parallel.  $Y_3$  is the admittance of the general circuit without the compensation circuitry.  $Y_3$  consists of  $R_a$  and  $Y_{m1+2}$  in series.  $Y_1$  in parallel with  $Y_{\text{comp}}$  forms  $Y_{\text{tbf}}$ . Before fusion occurs, the left branch of the general circuit with the admittance  $Y_2$  is not present, and the general circuit admittance reduces to  $Y_{tbf}$  (i.e., when  $G_p = 0$ ,  $Y_t = Y_{tbf}$ ).

the multiple sine wave method (MSW). The main problems facing both LN and MSW are phase offset and possible attenuation of current, caused by different pieces of equipment used in the experiment (amplifiers, filters, etc.).

PL requires compensation of  $R_a$  and  $C_{m1}$ , using the capacitance neutralization circuitry of a patch-clamp amplifier. Small changes in  $C_{\rm m1}$  can be measured from the out-of-phase output of a PSD, provided the experimenter has set the correct phase angle, but calculation of elements other than  $C_{\rm m1}$  is not available. The phase adjustment (Neher-Marty technique (NM); Neher and Marty, 1982) requires the operator to dither the amplifier's C-slow potentiometer while simultaneously changing the PSD phase angle. The correct phase angle is selected when C-slowinduced signals create maximum signals of the opposite sign in the PSD out-of-phase channel and no changes in the in-phase signal. PL also requires calibration of the out-ofphase channel, which is usually performed by displacing the C-slow potentiometer by a known value (a calibration "jump," which may require recalibration of the C-slow potentiometer). NM takes into account the phase offset introduced by equipment. Unfortunately, fluctuations of  $R_a$ and  $C_{\rm m1}$  can occur in the course of an experiment, requiring an additional adjustment of the phase angle, for example, with the phase tracking technique (Joshi and Fernandez, 1988).

Below we describe how C-slow potentiometer jumps can be used both for the determination of phase shift and attenuation caused by experimental equipment and, by successive application of MSW, for calculation of parameters  $R_{\rm a}$ ,  $G_{\rm m1}$ , and  $C_{\rm m1}$ . We demonstrate how these jumps can be employed for an automatic setting of the correct phase angle in accordance with the Neher-Marty technique. Furthermore, we show how MSW can be modified for calculations of fusion pore conductance and the parameters  $G_{\rm m2}$  and  $C_{\rm m2}$  of the second (fusing) membrane.

#### **MATERIALS AND METHODS**

GP64, the baculovirus envelope protein that mediates membrane fusion during infection, has recently been shown to mediate fusion with human red blood cells (Leikina and Chernomordik, personal communication). A stably transfected Sf9 cell line expressing GP64 was a kind gift of Dr. Gary Blissard. For whole-cell recordings, solutions were (concentration in mM) 111 KGlu, 13.8 KCl, 1.7 MgCl<sub>2</sub>, 8.5 HEPES, 4.3 EGTA, 43 sucrose, pH 7.2 with KOH, and pCa 8 with 0.71 mM CaCl<sub>2</sub> (internal), and 47.4 NaCl, 8.5 KCl, 3.4 CaCl<sub>2</sub>, 4.3 MgCl<sub>2</sub>, 4.3 glucose, 8.5 MES (2-[*N*-morpholino]ethanesulfonic acid; Sigma, St. Louis, MO), 163.8 sucrose, pH 6.2 with NaOH (external). Fusion was triggered with a highly buffered acidic solution (43 Na citrate, 43 NaCl, 8.5 KCl, 3.4 CaCl<sub>2</sub>, 4.3 MgCl<sub>2</sub>, 4.3 glucose, 17.2 sucrose, pH 5.0 with NaOH) delivered to Sf9/erythrocyte cell pairs by ejection from a micropipette. Patch pipettes had a resistance of 1–2 MΩ. The mean value of the access resistance shortly after the patch of membrane was ruptured was 4.6 MΩ.

TRAM was implemented in locally written software (*Browse*, available upon request). A 2-kHz, 50-mV peak-to-peak sine wave was superimposed on the holding potential (-20 mV). Sinusoidal current was filtered at 5 kHz with an 8-pole Bessel filter (Frequency Devices, Haverhill, MA), digitized at 40 kHz, and separated into in-phase and out-of-phase components (Joshi and Fernandez, 1988). Phase was set using automated NM (see

the Results for more details). Subsequently, the software output admittance,  $Y_{\rm out}$ , was calculated by dividing the current by the stimulus sine wave amplitude. The fusion pore conductance,  $G_{\rm p}$ , and the capacitance of the erythrocyte,  $C_{\rm m2}$ , were calculated on the basis of the fusion-induced increment of  $Y_{\rm out}$ ,  $(Y_{\rm incr})_{\rm out}$ :  $G_{\rm p}=(R^2+I^2)/[|T(\omega)|^2R]$  and  $C_{\rm m2}=(R^2+I^2)/[|T(\omega)|^2\omega I]$  (Lindau, 1991). In these equations, R and I are the real and the imaginary components of  $(Y_{\rm incr})_{\rm out}$ ,  $\omega=2\pi f$ , f is the frequency of the stimulating sine wave, and  $T(\omega)$  is the scaling factor.  $T(\omega)$  was calculated as  $1/(1+j\omega C_{\rm sl}/G_{\rm sr})$ , where  $C_{\rm sl}$  and  $G_{\rm sr}$  are settings of patch-clamp capacitance neutralization circuitry.

#### **RESULTS**

# Outputs of a phase-sensitive detector do not represent pure real and imaginary components of the pipette-cell admittance

Let us first discuss the factors defining the signals recorded from the in-phase and out-of-phase channels of a PSD. (Below we always represent the in-phase and out-of-phase PSD signals as real and imaginary components of the source admittance.) Although the following analysis is not new, it will be needed below. Let  $A_{\rm out}$  be the signal in-phase and  $B_{\rm out}$  be the signal out-of-phase with respect to a stimulus signal with an angular frequency  $\omega$ . Then the output complex admittance  $Y_{\rm out} = A_{\rm out} + jB_{\rm out}$ , where  $j = \sqrt{-1}$ . The total admittance  $Y_{\rm t} = A_{\rm t} + jB_{\rm t}$ , where  $A_{\rm t}$  is the real and  $B_{\rm t}$  is the imaginary component of  $Y_{\rm t}$  (Fig. 1). Let us analyze the relationship between  $Y_{\rm t}$  and  $Y_{\rm out}$ .

Equipment (amplifiers, filters, etc.) introduces phase shifts into the circuit admittance by the frequency-dependent angle  $\varphi_{\rm eq}$ , which results in the multiplication of  $Y_{\rm t}$  by  $\exp(j\varphi_{\rm eq})$ . For example, the 5-kHz 8-pole Bessel filter used in our experiments is the major phase-shifting source: it introduces  $\varphi_{\rm eq} = -72.3^{\circ}$  (f = 2 kHz). Besides shifting the phase, the equipment may also cause some attenuation of the source admittance, resulting in the multiplication of  $Y_{\rm t}$  by the constant K > 0. Thus the admittance at the input of the PSD,  $Y_{\rm in}$ , will be

$$Y_{\rm in} = A_{\rm in} + jB_{\rm in} = (A_{\rm t} + jB_{\rm t})\exp(j\varphi_{\rm eq})K. \tag{1}$$

The PSD usually has phase adjustment circuitry that allows a user to set the desirable phase angle. If we denote the phase shift introduced by a PSD as  $\varphi_{psd}$ , we have

$$Y_{\text{out}} = A_{\text{out}} + jB_{\text{out}} = (A_{\text{in}} + jB_{\text{in}})\exp(j\varphi_{\text{psd}}). \tag{2}$$

From Eqs. 1 and 2 we obtain

$$Y_{\text{out}} = (A_{\text{t}} + jB_{\text{t}}) \exp[j(\varphi_{\text{eq}} + \varphi_{\text{nsd}})]K. \tag{3}$$

Whereas  $\varphi_{psd}$  is known during the experiment,  $\varphi_{eq}$  and K have to be determined to use LN and MSW. These methods consider the "true" admittance of the general circuit (Fig. 1). More accurately, they deal with its right-hand part, consisting of  $R_{m1}$ ,  $C_{m1}$ ,  $E_r$ ,  $R_a$ ,  $C_{sl}$ , and  $G_{sr}$ .

# C-slow displacement can be used to restore the real and the imaginary components of the pipette and cell admittance to their true values

To calculate the compensation vector for restoration of  $Y_t$ , one can apply a signal with known amplitude and phase with respect to the stimulus sine wave (Donnelly, 1994). Below we consider the capacitance neutralization circuitry of a patch-clamp amplifier as a source of such a signal.

Suppose we are performing a "jump," increasing  $C_{\rm sl}$  by a value  $\Delta C_{\rm sl}$ , starting from arbitrary values of  $G_{\rm sr}$ ,  $C_{\rm sl}$ . The general circuit from Fig. 1 contains a combination of two membranes with the resulting admittance  $Y_3$  in parallel with the compensation circuitry having "negative" admittance  $Y_{\rm comp}$  (see Fig. 1 legend for more details). The admittance of the general circuit can thereby be represented as

$$Y_{t} = Y_{3} - Y_{\text{comp}}. \tag{4}$$

If  $G_p = 0$ , Eq. 4 can be rewritten as  $Y_{tbf} = Y_1 - Y_{comp}$  ( $Y_{tbf}$  and  $Y_1$  are defined in the legend to Fig. 1). When a jump is being issued,

$$Y1_t = Y_3 - Y1_{comp},$$
 (5)

where  $Y1_t$  is the new value of  $Y_t$ , and  $Y1_{comp}$  is the new admittance of the compensation circuitry. Assuming that  $Y_3$  does not change during the jump, from Eqs. 4 and 5 we easily obtain the next expression for the general circuit admittance change:

$$J = Y1_{t} - Y_{t} = Y_{comp} - Y1_{comp}.$$
 (6)

Since  $C_{\rm sl}$  is in series with  $G_{\rm sr}$ ,  $Y_{\rm comp}$  can be calculated using the next equation:

$$Y_{\text{comp}} = G_{\text{sr}} j \omega C_{\text{sl}} / (G_{\text{sr}} + j \omega C_{\text{sl}}). \tag{7}$$

Similarly, for the admittance  $Y1_{comp}$  we have

$$Y1_{\text{comp}} = G_{\text{s}}j\omega(C_{\text{sl}} + \Delta C_{\text{sl}})/[G_{\text{sr}} + j\omega(C_{\text{sl}} + \Delta C_{\text{sl}})].$$
 (8)

Substituting the expressions for  $Y_{\text{comp}}$  and  $Y_{\text{lcomp}}$  into Eq. 6, we obtain

$$J = -j\omega\Delta C_{\rm sl}/[(1+j\omega C_{\rm sl}/G_{\rm sr})(1+j\omega(C_{\rm sl}+\Delta C_{\rm sl})/G_{\rm sr})]. \tag{9}$$

In the course of an experiment, instead of J we actually measure the phase-shifted and attenuated value  $J_{\rm out}$ . According to the discussion above,  $J_{\rm out}$  can be calculated using the following equation:

$$J_{\text{out}} = Y_1 \exp[j(\varphi_{\text{eq}} + \varphi_{\text{psd}})]K - Y_1 \exp[j(\varphi_{\text{eq}} + \varphi_{\text{psd}})]K$$
$$= J \exp[j(\varphi_{\text{eq}} + \varphi_{\text{psd}})]K. \tag{10}$$

An important property of the jump-induced values J and  $J_{\rm out}$  is their independence from the membrane parameters of the

general circuit. The actual admittance change J depends only upon the magnitudes of  $G_{\rm SF}$ ,  $C_{\rm sl}$ , and  $\Delta C_{\rm sl}$ , whereas the experimental value  $J_{\rm out}$  is also determined by the characteristics of one's equipment ( $\varphi_{\rm eq}$  and K) and the preset phase shift  $\varphi_{\rm psd}$ .

Finally, the values of  $\varphi_{eq}$  and K can be calculated following the considerations below. From Eq. 10 we obtain

$$J_{\text{out}}/J = K \exp[j(\varphi_{\text{eq}} + \varphi_{\text{nsd}}))]. \tag{11}$$

Now if  $C_{\rm sl}$ ,  $G_{\rm sr}$ , and  $\varphi_{\rm psd}$  are known, we can compute J from Eq. 9 and, using the experimental value  $J_{\rm out}$ , calculate the attenuation K and equipment-originated phase shift  $\varphi_{\rm eq}$  from Eq. 11. As we can see, the right part of Eq. 11 is just the polar representation of the complex number  $Q = J_{\rm out}/J$ . Thus we have

$$K = |Q|,$$

$$\varphi_{eq} + \varphi_{psd} = \arg(Q),$$
(12)

and consequently,

$$\varphi_{\rm eq} = \arg(Q) - \varphi_{\rm psd}. \tag{13}$$

If Q = a + jb, we can use the next two expressions for arg(Q):

$$arg(Q) = arctan(b/a), if a > 0;$$

$$arg(Q) = arctan(b/a) + \pi$$
, if  $a < 0$ ;

$$arg(Q) = \pi/2$$
, if  $a = 0$  and  $b > 0$ ;

$$arg(O) = -\pi/2$$
, if  $a = 0$  and  $b < 0$ .

$$\begin{aligned} |J_{\text{out}}| &= |J \exp[j(\varphi_{\text{eq}} + \varphi_{\text{psd}})]K| = |J|K \\ &= K\omega\Delta C_{\text{sl}}/|(1 + j\omega C_{\text{sl}}/G_{\text{sr}})[1 + j\omega(C_{\text{sl}} + \Delta C_{\text{sl}})/G_{\text{sr}}]| \\ &= K\omega\Delta C_{\text{sl}}/([1 + (\omega C_{\text{sl}}/G_{\text{sr}})^{2}] \\ & \cdot \{1 + [\omega(C_{\text{sl}} + \Delta C_{\text{sl}})/G_{\text{sr}}]^{2}\})^{1/2}. \end{aligned}$$
 (F1)

From Eq. F1, assuming K = 1, we obtain

$$|J_{\text{out}}| = \omega \Delta C_{\text{sl}} / ([1 + (\omega C_{\text{sl}} / G_{\text{sr}})^2]$$

$$\cdot \{1 + [\omega (C_{\text{sl}} + \Delta C_{\text{sl}}) / G_{\text{sr}}]^2 \})^{1/2}.$$
(F2)

The last equation shows that the absolute value of the jump can easily be calculated when the compensation parameters  $C_{\rm sl}$  and  $G_{\rm sr}$  are known. This equation also allows us to understand the nature of "calibration attenuation." Indeed,  $|J_{\rm out}|$  is less than the expected calibration-related increment of  $Y_{\rm t}$ , which is equal to  $\omega\Delta C_{\rm sl}$ . Equation F2 reveals that the ratio  $\tau=|J_{\rm out}|/(\omega\Delta C_{\rm sl})$  is always less than 1, and the lapse between  $|J_{\rm out}|$  and  $\omega\Delta C_{\rm sl}$  increases with  $\omega$  and is negligible when  $\omega C_{\rm sl}$  is much less than  $G_{\rm sr}$ . For instance, for the compensation parameters  $C_{\rm sl}=23$  pF,  $\Delta C_{\rm sl}=1$  pF, and  $G_{\rm sr}=0.2$   $\mu\rm S$ ,  $\tau$  is equal to 0.979 for f=200 Hz and equals 0.647 for f=1000 Hz.

Knowledge of the distortion parameters K and  $\varphi_{\rm eq}$  allows us to transform the experimental admittance  $Y_{\rm out} = A_{\rm out} + jB_{\rm out}$  into the undistorted source admittance  $Y_{\rm t} = A_{\rm t} + jB_{\rm t}$ . Using Eq. 3, we can come to the following formula:

$$\begin{split} Y_{\rm t} &= A_{\rm t} + jB_{\rm t} = (A_{\rm out} + jB_{\rm out})/\{K \exp[j(\varphi_{\rm eq} + \varphi_{\rm psd})]\} \\ &= \exp[-j(\varphi_{\rm eq} + \varphi_{\rm psd})](A_{\rm out} + jB_{\rm out})/K. \end{split}$$

Or, if  $G_p = 0$ , the last equation can be rewritten as follows:

$$Y_{\text{tbf}} = \exp[-j(\varphi_{\text{eq}} + \varphi_{\text{psd}})](A_{\text{out}} + jB_{\text{out}})/K.$$

Compared to other techniques for restoration of the pipette and cell admittance, the above approach has obvious advantages. It does not require any additional hardware as suggested by Donelly (1994) or Rohlicek and Schmid (1994); nor does it need readjustments after filter or acquisition settings have been changed, such as those proposed by Gillis (1995).

## Use of the restored value of cell-pipette admittance in the multiple sine wave algorithm

As soon as  $Y_{\rm t}$  is restored, it becomes possible to obtain the true values of  $R_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$ . For this purpose let us use MSW (Rohlicek and Rohlicek, 1993; Donnelly, 1994). In this case the stimulus signal consists of at least two sine waves, and the assumption  $G_{\rm p}=0$  is made (i.e.,  $Y_{\rm t}=Y_{\rm tbf}$ ).

The calculation procedure looks as follows. Let  $\omega_1$  and  $\omega_2$  be frequencies incorporated in the stimulus signal, and  $A_{\rm tbf}(\omega_1) + jB_{\rm tbf}(\omega_1)$  and  $A_{\rm tbf}(\omega_2) + jB_{\rm tbf}(\omega_2)$  be the admittances of the general circuit corresponding to those frequencies. Using Eq. 4 in its rewritten form, and Eq. 7, we obtain

$$Y_1(\omega_k) = A_{\text{thf}}(\omega_k) + jB_{\text{thf}}(\omega_k) + Y_{\text{comp}}$$
 (14)

and

$$Y_1(\omega_k) = A_{tbf}(\omega_k) + jB_{tbf}(\omega_k) + G_{sr}j\omega_k C_{sl}/(G_{sr} + j\omega_k C_{sl}),$$

where k=1, 2 and  $Y_1(\omega_k)$  is the admittance of the first (voltage-clamped) membrane circuitry, containing  $R_{\rm a}$ ,  $C_{\rm m1}$ ,  $G_{\rm m1}$ , and  $E_{\rm r}$ .

Now denoting  $c_k + jd_k = Y_1(\omega_k)$ , we can use the following generalizations of formulae from Donnelly (1994) to calculate  $R_a$ ,  $C_{m1}$ , and  $G_{m1}$ :

$$R_{a} = (\omega_{1} - \omega_{2})/$$

$$[(d_{1} - d_{2})(d_{1}\omega_{1} - d_{2}\omega_{2})/(c_{1} - c_{2}) + c_{1}\omega_{1} - c_{2}\omega_{2}],$$
(15)

$$G_{\rm m1} = (c_1 d_2 \omega_2 - c_2 d_1 \omega_1) /$$

$$[d_2 \omega_2 - d_1 \omega_1 - R_a (c_1 d_2 \omega_2 - c_2 d_1 \omega_1)],$$
(16)

$$C_{\rm m1} = (c_2 - c_1)/ \tag{17}$$

$$\{R_a[d_2\omega_2-d_1\omega_1-R_a(c_1d_2\omega_2-c_2d_1\omega_1)]\}.$$

 $<sup>^{1}</sup>$ Let us consider the absolute value of  $J_{\rm out}$ , which is used in the PL technique for the computation of small changes in the cell capacitance  $C_{\rm m1}$ . From Eq. 10 we can derive

It is necessary to note that in Eqs. 15–17,  $\omega_1 \neq 0$ ,  $\omega_2 \neq 0$ , and  $\omega_1 \neq \omega_2$ . However, these equations can be used even if one of the frequencies equals zero. In this case we should additionally assume that the resting potential is known and constant (see Gillis, 1995, for more details).

If both frequencies are greater than zero, Eqs. 15 and 16 can be used for the calculation of  $E_{\rm r}$ . If several sine waves in the stimulus signal are superimposed on the holding potential  $(U_{\rm h})$ , and  $I_{\rm h}$  is the current generated by an amplifier to clamp the first membrane to  $U_{\rm h}$ , we can easily obtain the following expression:

$$I_{h}R_{a} + I_{h}R_{m1} + E_{r} = U_{h}$$

where  $R_{\rm m1}=1/G_{\rm m1}$ . Now employing the value  $G_{\rm DC}=I_{\rm h}/U_{\rm h}$ , we have  $G_{\rm DC}U_{\rm h}(R_{\rm a}+R_{\rm m1})+E_{\rm r}=U_{\rm h}$ , and hence

$$E_{\rm r} = U_{\rm h} [1 - G_{\rm DC}(R_{\rm a} + 1/G_{\rm m1})], \tag{18}$$

where  $R_{\rm a}$  and  $G_{\rm m1}$  are found from Eqs. 15 and 16 ( $\omega_1 \neq 0$  and  $\omega_2 \neq 0$ ).

The advantage of MSW is the ability to calculate  $C_{\rm m1}$  at times when  $R_a$  and  $G_{m1}$  are also changing. Compared to the "phase tracking" technique (Joshi and Fernandez, 1988), which adjusts admittance measurement according to changes in  $R_a$ , MSW does not require any additional on-line actions, is free from deficiencies of phase tracking related to pipette capacitance (Gillis, 1995), and yields more information about cell parameters  $(E_r, R_a, \text{ and } G_{m1})$ . Although dual-frequency methods are generally noisier than singlefrequency techniques (Gillis, 1995), some new statistical approaches to reducing the estimation errors of the cell parameters were recently introduced (Barnett and Misler, 1997). We have shown that MSW can be improved by the incorporation of a new phase setting procedure, introduced in the previous section, yielding values of the pipette and cell admittance previously distorted by equipment.<sup>2</sup>

<sup>2</sup>MSW and the new phase setting procedure rely on readings of the C-slow and G-series potentiometers, for determination of  $Y_{\rm comp}$  and  $\varphi_{\rm eq}$ . These readings may be inaccurate (on the order of 5–10%) and introduce computational errors. Empirically, we found accurate calibration of these potentiometers to be of questionable value, because there was a nonlinear interaction between them, requiring the compilation of a look-up table containing all combinations of settings. Instead, we propose the following two-stage, "off-line playback" method to determine  $Y_{\rm comp}$  and  $\varphi_{\rm eq}$  when more accurate measurements are required. This procedure may also be used to accurately measure values of  $C_{\rm sl}$  and  $G_{\rm sr}$ . First, the value  $\varphi_{\rm eq}$  is determined independently of  $C_{\rm sl}$  and  $G_{\rm sr}$  readings. For this purpose, the amplifier head-stage input is open to the air (disconnected), stray capacitance is compensated with the C-fast circuitry, and  $\Delta C_{\rm sl}$  is applied with settings of  $C_{\rm sl}$  and  $G_{\rm sr}$  that satisfy the following conditions:  $\omega C_{\rm sl} \ll G_{\rm sr}$ ,  $\omega \Delta C_{\rm sl} \ll G_{\rm sr}$ . Then we can derive  $J = -j\omega \Delta C_{\rm sl}$  from Eq. 9, and obtain from Eq. 13

$$arphi_{
m eq} = {
m arg}(J_{
m out}/J) - arphi_{
m psd} = {
m arg}(J_{
m out}) - {
m arg}(J) - arphi_{
m psd}$$

$$= {
m arg}(J_{
m out}) + 3\,\pi/2 - arphi_{
m psd}.$$

It is easy to see that this expression for  $\varphi_{\rm eq}$  does not depend on the actual values  $C_{\rm sl}$  and  $G_{\rm sr}$ . For the second stage of this off-line playback procedure, the  $Y_{\rm comp}$  value equal to the one produced by the compensation circuitry during each real experiment should be measured. For this purpose, the  $C_{\rm sl}$ 

# Proof of the validity of the empirical phase adjustments used in the "piecewise-linear" technique

Let us first derive the equation for the PL phase angle,  $\varphi_{\rm eq} + \varphi_{\rm psd}$ , which we denote as  $\varphi$ . Only  $\varphi_{\rm psd}$  is experimentally adjustable, whereas  $\varphi_{\rm eq}$  is constant. We assume that the attenuation caused by equipment is negligible, so K=1. When  $G_{\rm p}=0$ , we can rewrite Eq. 3:

$$Y_{\text{out}} = A_{\text{out}} + jB_{\text{out}} = (A_{\text{tbf}} + jB_{\text{tbf}})\exp(j\varphi). \tag{19}$$

According to Lindau and Neher (1988), in this case a small change in  $C_{\rm m1}$ ,  $\Delta C_{\rm m1}$  induces the following alteration  $\Delta Y_{\rm tbf}$  in  $Y_{\rm tbf}$ :

$$\Delta Y_{\rm tbf} = T(\omega)^2 (A_{\rm tbf} + j\omega \Delta C_{\rm m1}). \tag{20}$$

Equation 20 holds true only if  $G_{\rm a} \gg G_{\rm m1}$  and  $G_{\rm a} \approx \omega C_{\rm m1}$  or  $G_{\rm a} > \omega C_{\rm m1}$  (the sign  $\approx$  in the last condition means "is comparable"). The factor  $T(\omega)$  is calculated according to (Lindau, 1991)

$$T(\omega) = 1/(1 + G_{\rm ml}/G_{\rm a} + j\omega C_{\rm ml}/G_{\rm a}).$$
 (21)

If  $G_{\rm m1} \ll G_{\rm a}$ ,

$$T(\omega) = 1/(1 + j\omega C_{\rm m1}/G_{\rm s}).$$
 (22)

Let  $\Delta Y_{\rm out}(\varphi)$  be the change in the output signal corresponding to a  $\Delta C_{\rm m1}$ -induced change  $\Delta Y_{\rm tbf}$  in the admittance  $Y_{\rm tbf}$ . We assume that the measurements are performed with the given phase  $\varphi$ . Then, from Eqs. 19 and 20,

$$\Delta Y_{\rm out}(\varphi) = \Delta Y_{\rm thf} \exp(j\varphi) = T(\omega)^2 (A_{\rm thf} + j\omega \Delta C_{\rm m1}) \exp(j\varphi).$$

Let  $\phi$  be the argument of  $T(\omega)$ . Then  $T(\omega) = |T(\omega)| \exp(j\phi)$ , and we can rewrite the previous expression for  $\Delta Y_{\text{out}}(\varphi)$ :

$$\Delta Y_{\text{out}}(\varphi) = |T(\omega)|^2 \exp[j(2\phi + \varphi)](A_{\text{tbf}} + j\omega\Delta C_{\text{ml}}). \tag{23}$$

The PL technique assumes that the phase  $\varphi$  is chosen such that

$$2\phi + \varphi = 0 \tag{24}$$

(Lindau and Neher, 1988). Indeed, denoting the phase  $\varphi$  (satisfying Eq. 24) as  $\varphi_{\rm PL}$ ,

$$\varphi_{\rm PL} = -2\phi = -2\arg(T(\omega)) = -2\arg[1/(1+j\omega C_{\rm ml}/G_{\rm a})].$$
(25)

and  $G_{\rm sr}$  potentiometers should be set to the values used in the experiment, and  $\varphi_{\rm psd}$  should be set at  $\varphi_{\rm psd} = -\varphi_{\rm eq}$ , where  $\varphi_{\rm eq}$  has been found as described here (we assume that the attenuation caused by equipment is negligible, so K=1). Then the increment of the circuit admittance  $Y_{\rm comp}$  should be measured by switching the C-slow circuitry off. When the input to the head stage of an amplifier is open, it is connected only to a "negative" admittance formed by the C-slow circuitry (see Fig. 1). Finally, this measured value  $Y_{\rm comp}$  is used in Eq. 14 to find the value  $Y_{\rm 1}(\omega_{\rm k})$  for Eqs. 15–17. Knowledge of  $Y_{\rm comp}$  allows one to calculate accurate values of  $C_{\rm sl}$  and  $G_{\rm sr}$ , using formulae similar to Eqs. 51 and 52 described below. Note that MSW does not require the actual magnitudes of  $C_{\rm sl}$  and  $G_{\rm sr}$ , because only  $Y_{\rm comp}$  is utilized in Eq. 14.

From Eq. 23,  $\Delta Y_{\rm out}(\varphi) = |T(\omega)|^2 (A_{\rm tbf} + j\omega\Delta C_{\rm m1})$ , and for the imaginary part of  $\Delta Y_{\rm out}(\varphi)$  we have  ${\rm Im}(\Delta Y_{\rm out}(\varphi)) = |T(\omega)|^2 \omega \Delta C_{\rm m1}$ . Thus small changes in the imaginary part of the admittance are recorded from the out-of-phase channel of a PSD. Notice that  $\varphi_{\rm PL}$  is determined by the cell "circuitry," but not by compensation parameters.

Now, having derived Eq. 25 for the phase angle used in PL, we will describe how the phase is actually set in NM and investigate when this value of  $\varphi$  satisfies Eq. 24. We first derive an equation describing changes in the general admittance induced by displacements of the potentiometer C-slow. Using Eqs. 9 and 10 and assuming K = 1, we obtain the following formula:

$$J_{\text{out}} = (-j\omega\Delta C_{\text{sl}}/\{(1+j\omega C_{\text{sl}}/G_{\text{sr}})[1+j\omega(C_{\text{sl}}+\Delta C_{\text{sl}})/G_{\text{sr}}]\})$$

 $\cdot \exp(j\varphi)$ . (26)

The phase in NM is set to allow  $J_{\rm out}$  to be represented by a vector going downward along the imaginary axis, which means that  $\arg(J_{\rm out})=3\pi/2$ . Let us denote this phase value as  $\varphi_{\rm NM}$ . Because multiplication (division) of complex numbers adds (subtracts) their arguments, we have from Eq. 26

$$3\pi/2 = \arg(J_{\text{out}}) = \arg(-j\omega\Delta C_{\text{sl}}) + \varphi_{\text{NM}} -$$

$$\arg\{(1 + j\omega C_{\text{sl}}/G_{\text{sr}})[1 + j\omega(C_{\text{sl}} + \Delta C_{\text{sl}})/G_{\text{sr}}]\}.$$

Because  $\arg(-j\omega\Delta C_{\rm sl}) = 3\pi/2$ , we obtain from the previous equation

$$\varphi_{\text{NM}} = \arg(1 + j\omega C_{\text{sl}}/G_{\text{sr}}) + \arg[1 + j\omega(C_{\text{sl}} + \Delta C_{\text{sl}})/G_{\text{sr}}]. \tag{27}$$

Note that unlike  $\varphi_{PL}$ ,  $\varphi_{NM}$  is determined by elements of the neutralization circuitry.

Now we can determine if  $\varphi_{\rm NM}$  satisfies Eq. 24. In addition to earlier assumptions that  $(G_{\rm a}\gg G_{\rm m1},\,G_{\rm a}>\omega C_{\rm m1})$  or  $G_{\rm a}\approx\omega C_{\rm m1}$ ), we will need the next three:  $C_{\rm sl}\cong C_{\rm m1},\,G_{\rm sr}\cong G_{\rm a}$ , and  $\Delta C_{\rm sl}\ll C_{\rm sl}$ . Based on all of these assumptions, we have from Eq. 27

$$\varphi_{\text{NM}} = 2 \arg(1 + j\omega C_{\text{sl}}/G_{\text{sr}}) = 2 \arctan(\omega C_{\text{sl}}/G_{\text{sr}})$$
$$= 2 \arctan(\omega C_{\text{ml}}/G_{\text{a}}). \tag{28}$$

From Eq. 25 we can easily obtain  $\varphi_{PI}$ :

$$\varphi_{\rm PL} = -2 \arg[1/(1 + j\omega C_{\rm m1}/G_{\rm a})] = 2 \arg(1 + j\omega C_{\rm m1}/G_{\rm a})$$

$$= 2 \arctan(\omega C_{\rm m1}/G_{\rm a}) = \varphi_{\rm NM}.$$

This proves our point: when  $\varphi = \varphi_{\rm NM}$ , the phase angle satisfies the requirement of PL. This statement holds true as long as  $C_{\rm sl} \cong C_{\rm m1}$  and  $G_{\rm sr} \cong G_{\rm a}$ . The assumptions  $C_{\rm sl} \cong C_{\rm m1}$  and  $G_{\rm sr} \cong G_{\rm a}$  are true immediately after the compensation has been performed, because it is possible to show that for a single cell the compensation parameters satisfy the next two equations (Lindau and Neher, 1988):

$$C_{\rm sl} \cong C_{\rm m1}(1 - 2R_{\rm a}/R_{\rm m1}),$$
 (29)

$$G_{\rm sr} \cong G_{\rm a}(1 - R_{\rm a}/R_{\rm m1}).$$
 (30)

If  $G_a \gg G_{\rm m1}$ , we can easily produce the necessary conditions  $C_{\rm sl} \cong C_{\rm m1}$  and  $G_{\rm sr} \cong G_{\rm a}$  from Eqs. 29 and 30.

This analytical proof of the validity of the empirical phase adjustments used in PL replaces the assertion that  $C_{\rm sl}$  is equivalent to  $C_{\rm m1}$  (Lindau and Neher, 1988).

Finally, using Eq. 10, we introduce an algorithm for quickly determining the PSD phase shift according to NM. Suppose a calibration jump routinely used in NM to determine the scaling factor  $|T(\omega)|^2$  is made when  $\varphi_{\rm psd} = \varphi_{\rm NM}$ . Assuming K=1, from Eq. 10 we have:

$$J_{\text{out}} = J \exp[j(\varphi_{\text{eq}} + \varphi_{\text{NM}})],$$

and

$$3\pi/2 = \arg(J_{\text{out}}) = \arg(J) + \varphi_{\text{eq}} + \varphi_{\text{NM}}. \tag{30a}$$

If the same jump is made when  $\varphi_{psd} = \varphi 1_{psd} \neq \varphi_{NM}$ , where  $\varphi 1_{psd}$  is known and the output signal  $J1_{out}$  is measured, we have

$$J1_{\text{out}} = J \exp[j(\varphi_{\text{eq}} + \varphi 1_{\text{psd}})]$$

and

$$arg(J1_{out}) = arg(J) + \varphi_{eq} + \varphi 1_{psd}$$
.

Subtracting the last equation from Eq. 30a, we have  $3\pi/2 = \arg(J1_{\rm out}) = \varphi_{\rm NM} - \varphi 1_{\rm psd}$ . This gives us the next simple expression, which can be implemented in the software to adjust phase according to NM:

$$\varphi_{\text{NM}} = \varphi 1_{\text{psd}} + 3\pi/2 - \arg(J1_{\text{out}}).$$

Thus a calibration jump can yield the phase. The dithering of potentiometers required in NM can be completely avoided, saving valuable time in the course of an experiment. Note that the described algorithm for finding  $\varphi_{\rm NM}$  is independent of the readings of the compensation potentiometers (see footnote 2).

## Fusion pore calculation algorithms: initial considerations

We will now describe some algorithms for calculation of elements of an equivalent circuit for membrane fusion (Fig. 1), primarily, the fusion pore conductance. The fusing circuitry consists of  $G_{\rm p}$ ,  $C_{\rm m2}$ , and  $G_{\rm m2}$ , and has admittance  $Y_2$ . Therefore, calculation of  $G_{\rm p}$ ,  $C_{\rm m2}$ , and  $G_{\rm m2}$  is based on the determination of  $Y_2$ .

Before fusion, there is no electrical contact between the two membrane systems ( $G_{\rm p}=0$ ), so the circuit is represented by  $R_{\rm a}$ ,  $C_{\rm m1}$ ,  $G_{\rm m1}$ ,  $E_{\rm r}$ ,  $C_{\rm sl}$ , and  $G_{\rm sr}$  and has admittance  $Y_{\rm tbf}$ . The in-phase and out-of-phase signals from a PSD at this time comprise the base line and are usually subtracted from the admittance of the general circuit when fusion has occurred ( $G_{\rm p}>0$ ). Let us assume, during the entire process of fusion pore formation, constancy of the parameters  $R_{\rm a}$ ,  $G_{\rm m1}$ , and  $G_{\rm m1}$  of the patch-clamped membrane, and of the compensation parameters  $C_{\rm sl}$  and  $G_{\rm sr}$ . Unlike Lindau (1991), we will not require either  $C_{\rm m2}$  or  $G_{\rm m2}$  to be constant.

Now for the base line admittance we obtain, using the rewritten form of Eq. 4,

$$Y_{\text{tbf}} = Y_1 - Y_{\text{comp}} = G_a Y_{\text{ml}} / (G_a + Y_{\text{ml}}) - Y_{\text{comp}},$$
 (31)

where  $Y_{\rm m1}$  denotes the admittance of the patch-clamped membrane with elements  $C_{\rm m1}$ ,  $G_{\rm m1}$ , and  $E_{\rm r}$ ;  $Y_{\rm 1}$  stands for the resulting admittance of  $Y_{\rm m1}$  and  $R_{\rm a}$  in series. If we use sine wave stimulation with the frequency  $\omega>0$ , we can ignore the resting potential  $E_{\rm r}$ . In this case  $Y_{\rm m1}=G_{\rm m1}+j\omega C_{\rm m1}$ , because this part of the general circuit contains just  $C_{\rm m1}$  and  $G_{\rm m1}$  in parallel. Let us denote  $Y_{\rm m2}=G_{\rm m2}+j\omega C_{\rm m2}$  as the admittance of the second membrane, and  $Y_{\rm 2}$  as the resulting admittance of  $Y_{\rm m2}$  and  $G_{\rm p}$  in series. As in Eq. 31, the total admittance  $Y_{\rm t}$  of the circuit in the general case  $(G_{\rm p}>0)$  can be written as

$$Y_{\rm t} = Y_3 - Y_{\rm comp} = G_{\rm a} Y_{\rm ml+2} / (G_{\rm a} + Y_{\rm ml+2}) - Y_{\rm comp},$$
(32)

where  $Y_3$  includes  $R_a$  and  $Y_{m1+2}$  in series, and  $Y_{m1+2} = Y_{m1} + Y_2$ , because  $Y_{m1}$  is in parallel with  $Y_2$ . Now from Eqs. 31 and 32, the fusion-related increment of the total admittance,  $Y_{incre}$ , is

$$Y_{\text{incr}} = Y_{\text{t}} - Y_{\text{tbf}}$$

$$= (Y_3 - Y_{\text{comp}}) - (Y_1 - Y_{\text{comp}})$$

$$= Y_3 - Y_1$$

$$= G_{\text{a}}Y_{\text{m1+2}}/(G_{\text{a}} + Y_{\text{m1+2}}) - G_{\text{a}}Y_{\text{m1}}/(G_{\text{a}} + Y_{\text{m1}}).$$
(33)

Substituting  $Y_{\rm m1}$  with  $G_{\rm m1} + j\omega C_{\rm m1}$ , we obtain from Eq. 33

$$Y_{\text{incr}} = G_{\text{a}}(G_{\text{m1}} + j\omega C_{\text{m1}} + Y_2)/(G_{\text{a}} + G_{\text{m1}} + j\omega C_{\text{m1}} + Y_2)$$
$$- G_{\text{a}}(G_{\text{m1}} + j\omega C_{\text{m1}})/(G_{\text{a}} + G_{\text{m1}} + j\omega C_{\text{m1}})$$

or

$$Y_{\text{incr}} = T(\omega)Y_2/[1/T(\omega) + Y_2/G_a],$$
 (34)

where  $T(\omega)$  is defined in Eq. 21.

Because  $Y_2$  is a series sequence of  $G_p$  and parallel combination of  $G_{\rm m2}$  and  $C_{\rm m2}$ ,

$$Y_2 = G_{\rm n}(G_{\rm m2} + j\omega C_{\rm m2})/(G_{\rm n} + G_{\rm m2} + j\omega C_{\rm m2}).$$
 (35)

The expression above leads to the following inequality<sup>3</sup>:

$$|Y_2/G_a| \le \min(G_p/G_a, |(G_{m2} + j\omega C_{m2})/G_a|).$$
 (36)

<sup>3</sup>From Eq. 35 we have

$$Y_2/G_a = (G_p/G_a)(G_{m2} + j\omega C_{m2})/(G_p + G_{m2} + j\omega C_{m2})$$

and

$$|Y_2/G_a| = |(G_p/G_a)(G_{m2} + j\omega C_{m2})/(G_p + G_{m2} + j\omega C_{m2})|$$
  
=  $(G_p/G_a)|(G_{m2} + j\omega C_{m2})/(G_p + G_{m2} + j\omega C_{m2})|$ .

From Eq. 36 it can easily be seen that we can neglect the ratio  $Y_2/G_a$  in Eq. 34, if any of the next two conditions is true:  $G_p \ll G_a$ , or  $G_{m2} \ll G_a$  and  $\omega C_{m2} \ll G_a$ . If this is the case, we can simplify the expression for  $Y_{incr}$ , given in Eq. 34:

$$Y_{\text{incr}} = T(\omega)^2 Y_2. \tag{37}$$

From Eq. 3 one can see that in the course of an experiment with the phase shift  $\varphi = \varphi_{\rm eq} + \varphi_{\rm psd}$  and attenuation K, the measured fusion-induced increment of the total admittance,  $(Y_{\rm incr})_{\rm out}$ , is equal to

$$(Y_{\text{incr}})_{\text{out}} = KY_{\text{t}} \exp(j\varphi) - KY_{\text{tbf}} \exp(j\varphi)$$
$$= K(Y_{\text{t}} - Y_{\text{tbf}}) \exp(j\varphi) = Y_{\text{incr}} \exp(j\varphi)K.$$

Representing  $T(\omega) = |T(\omega)| \exp(j\phi)$ , where  $\phi = \arg(T(\omega))$ , using Eq. 37, and assuming that either K = 1 or a corresponding correction has been performed according to Eq. 12, we have

$$(Y_{\text{incr}})_{\text{out}} = T(\omega)^2 Y_2 \exp(j\varphi) = |T(\omega)|^2 \exp(2j\phi) Y_2 \exp(j\varphi)$$
$$= Y_2 |T(\omega)|^2 \exp[j(\varphi + 2\phi)].$$

From the last equation we obtain

$$Y_2 = (Y_{\text{incr}})_{\text{out}} \exp[-j(\varphi + 2\phi)]/|T(\omega)|^2. \tag{38}$$

We have derived an important equation, showing the relationship between the fusion-related signal  $(Y_{incr})_{out}$  from the PSD and the admittance of the fusing circuitry, containing  $G_{\rm p}$ ,  $G_{\rm m2}$ , and  $C_{\rm m2}$ .

Below we describe two different approaches that both allow reconstruction of  $Y_2$  from Eq. 38 and its subsequent utilization for finding  $G_{\rm p}$ ,  $G_{\rm m2}$ , and  $C_{\rm m2}$ , which determine  $Y_2$  according to Eq. 35. In the first reconstruction approach, the phase is directly calculated using C-slow jumps. The experimenter performs a C-slow displacement and then triggers fusion. For further calculation during off-line analysis, one should use a part of the base line immediately preceding fusion. Exploiting the algorithm described above, the magnitudes of  $\varphi_{\rm eq}$ ,  $\varphi$ , and  $T(\omega)$ -determining parameters  $R_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$  can be obtained from Eqs. 13, and 15–17.

 $|Y_2/G_a| \le G_p/G_a$ , because  $|(G_{m2} + j\omega C_{m2})/(G_p + G_{m2} + j\omega C_{m2})|$  is equal to 1 when  $G_p = 0$  and is less than 1 when  $G_p > 0$ . Similarly, we obtain from Eq. 35,

$$Y_2/G_a = [(G_{m2} + j\omega C_{m2})/G_a]G_p/(G_p + G_{m2} + j\omega C_{m2})$$

and

$$|Y_2/G_a| = |(G_{m2} + j\omega C_{m2})/G_a||G_p/(G_p + G_{m2} + j\omega C_{m2})|$$
  

$$\leq |(G_{m2} + j\omega C_{m2})/G_a|,$$

because

$$|G_{\rm p}/(G_{\rm p} + G_{\rm m2} + j\omega C_{\rm m2})| \le 1.$$

Thus we have:  $|Y_2/G_a| \le \min(G_p/G_a, |(G_{m2} + j\omega C_{m2})/G_a|))$ .

Then one can calculate  $T(\omega)$  from Eq. 21, and find  $\phi = \arg[T(\omega)]$  and the factor  $\exp[-j(\varphi+2\phi)]/|T(\omega)|^2$ . Because, according to our previous assumptions,  $R_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$  do not change after fusion occurs, the experimenter then can restore  $Y_2$  from the distorted signal  $(Y_{\rm incr})_{\rm out}$ , using Eq. 38.

The second approach assumes that the phase has been set using the manual or automated Neher-Marty technique (i.e.,  $\varphi = -2\phi$ ). In this case we can conclude from Eqs. 38 and 24:

$$Y_2 = (Y_{\text{incr}})_{\text{out}} / |T(\omega)|^2. \tag{39}$$

The second approach is more "demanding": it requires correct compensation of  $G_a$  and  $C_{m1}$  for the entire experiment, whereas the first approach implies only the constancy of these parameters. If  $R_a$  or  $C_{m1}$  has changed and reached a new level between fusion triggering and pore formation (which is quite possible, because a delay between triggering and fusion can be as long as 217 s; Zimmerberg et al., 1994), only the first approach gives the correct results. Therefore, the first approach is "safer." However, the calculations in the second case are less complicated, because only  $|T(\omega)|^2$  has to be found for the restoration of  $Y_2$ .<sup>4</sup> Another advantage of the second approach is its independence from the readings of the compensation potentiometers, which can introduce additional errors (see footnote 2).

Both  $Y_2$  reconstruction approaches discussed in this section provide us with at least one equation of the type

$$Y_2 = A_2 + jB_2 (40)$$

<sup>4</sup>To calculate  $Y_2$  according to Eq. 39, the conditions  $C_{\rm sl} \cong C_{\rm ml}$ ,  $G_{\rm sr} \cong G_{\rm a}$ , and  $\Delta C_{\rm sl} \ll C_{\rm sl}$  have to be satisfied. Then the coefficient  $|T(\omega)|^2$  can be calculated from Eq. 22, using the approximations of  $G_{\rm a}$  and  $C_{\rm ml}$  by the compensation parameters  $G_{\rm sr}$  and  $C_{\rm sl}$ :

$$|T(\omega)|^2 = |1/(1 + j\omega C_{\rm ml}/G_{\rm a})|^2$$

$$= 1/[1 + (\omega C_{\rm ml}/G_{\rm a})^2]$$

$$= 1/[1 + (\omega C_{\rm sl}/G_{\rm sr})^2].$$
(F3)

If the compensation parameters are unknown,  $|T(\omega)|^2$  can still be found. Because  $\Delta C_{\rm sl} \ll C_{\rm sl}$ , we obtain from Eq. F2 (footnote 1)

$$|J_{\rm out}|/(\omega\Delta C_{\rm sl})$$

$$= 1/([1 + (\omega C_{sl}/G_{sr})^{2}]\{1 + [\omega(C_{sl} + \Delta C_{sl})/G_{sr}]^{2}\})^{1/2}$$

$$\approx 1/\{[1 + (\omega C_{sl}/G_{sr})^{2}]^{2}\}^{1/2} = 1/[1 + (\omega C_{sl}/G_{sr})^{2}] \quad (F4)$$

$$= |T(\omega)|^{2}.$$

Thus  $|T(\omega)|^2$  can be approximated by the ratio of absolute values of experimental and expected calibration jumps, provided that the calibration took place when  $G_{\rm p}=0$ , and that necessary assumptions  $(G_{\rm a}\gg G_{\rm ml},\ G_{\rm a}>\omega C_{\rm ml}$  or  $G_{\rm a}\approx\omega C_{\rm ml},\ C_{\rm sl}\cong C_{\rm ml},\ G_{\rm a}\cong G_{\rm a}$ , and  $\Delta C_{\rm sl}\ll C_{\rm sl})$  were all true. Furthermore, for the truthfulness of Eq. 39, either the condition  $G_{\rm p}\ll G_{\rm a}$  or both conditions,  $G_{\rm m2}\ll G_{\rm a}$  and  $\omega C_{\rm m2}\ll G_{\rm a}$ , should be satisfied. If we additionally require that  $\omega C_{\rm ml}\ll G_{\rm a}$ , we can approximate  $|T(\omega)|^2$  as 1, and Eq. 39 can be further simplified:

$$Y_2 = (Y_{\text{incr}})_{\text{out}}. (F5)$$

where  $A_2$  and  $B_2$  are known and can be used to construct and resolve the equations with respect to  $G_{\rm p}$  and, in some cases, with respect to parameters  $C_{\rm m2}$  and  $G_{\rm m2}$  of the second membrane. In the next section we will examine different algorithms for the calculation of these parameters. Each of the algorithms will be based on specific assumptions and sets of in-phase and out-of-phase signals obtained from a PSD.

# Using the modified multiple sine wave approach for calculation of the parameters of the second membrane

Although MSW was initially designed for the analysis of the first membrane ( $G_{\rm p}$  was neglected), it can be used for calculations of parameters included in the fusing circuitry, comprising  $G_{\rm p}$ ,  $C_{\rm m2}$ , and  $G_{\rm m2}$ . Using a stimulus signal containing two different sine waves with frequencies  $\omega_1 \neq \omega_2$ , we will try to determine all three parameters,  $G_{\rm p}$ ,  $G_{\rm m2}$ , and  $C_{\rm m2}$ . In this case Eq. 40 can be rewritten as follows:

$$Y_2(\omega_k) = A_2(\omega_k) + jB_2(\omega_k), \qquad k = 1, 2.$$
 (41)

Here we actually have the classical Lindau-Neher model, where instead of the circuitry containing  $G_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$ , a functionally identical circuitry with the corresponding parameters  $G_{\rm p}$ ,  $C_{\rm m2}$ , and  $G_{\rm m2}$  becomes apparent. Using Eqs. 15–17, making the substitutions  $G_{\rm p} \to G_{\rm a}$ ,  $G_{\rm m2} \to G_{\rm m1}$ ,  $C_{\rm m2} \to C_{\rm m1}$ , and denoting  $A_2(\omega_{\rm k}) = g_{\rm k}$  and  $B_2(\omega_{\rm k}) = h_{\rm k}$  (k=1,2), we obtain

$$G_{p} = [(h_{1} - h_{2})(h_{1}\omega_{1} - h_{2}\omega_{2})/(g_{1} - g_{2}) + g_{1}\omega_{1}$$
$$-g_{2}\omega_{2}]/(\omega_{1} - \omega_{2}), \tag{42}$$

$$G_{m2} = (g_1 h_2 \omega_2 - g_2 h_1 \omega_1) / [h_2 \omega_2 - h_1 \omega_1 - (g_1 h_2 \omega_2 - g_2 h_1 \omega_1) / G_n],$$
(43)

$$C_{\text{m2}} = (g_2 - g_1)G_{\text{p}}$$

$$/[h_2\omega_2 - h_1\omega_1 - (g_1h_2\omega_2 - g_2h_1\omega_1)/G_{\text{p}}].$$
(44)

If the first approach for  $Y_2(\omega_k)$  reconstruction is applied as described in the previous section,  $g_k$  and  $h_k$  are calculated from the PSD signals, and therefore  $G_p$ ,  $G_{m2}$ , and  $C_{m2}$  can be determined.

Let us consider the phase adjusted with the Neher-Marty technique as described above. We will denote the real and imaginary parts of the measured admittance increment  $(Y_{incr})_{out}$  for the frequency  $\omega_k$  as  $R_k$  and  $I_k$ , so that

$$[Y_{\text{incr}}(\omega_{\mathbf{k}})]_{\text{out}} = R_{\mathbf{k}} + jI_{\mathbf{k}}, \tag{45}$$

where k = 1, 2. In this case, according to Eq. 39,

$$g_{\nu} = R_{\nu}/|T(\omega_{\nu})|^2, \qquad h_{\nu} = I_{\nu}/|T(\omega_{\nu})|^2, \tag{46}$$

where  $|T(\omega_k)|^2$  can be found by employing Eqs. F3 or F4 from footnote 4. Substituting the values  $g_k$  and  $h_k$  from Eq.

46 into Eqs. 42–44, we can calculate  $G_{\rm p}$ ,  $C_{\rm m2}$ , and  $G_{\rm m2}$ . No matter which reconstruction approach is used, this modified MSW yields  $G_{\rm m2}$ , which has been neglected by previously published methods.

## Single sine wave approach: calculation of $G_p$ and $C_{m2}$ using two channels of a PSD

Now let us consider a single sine wave of frequency  $\omega$  contained in the stimulus signal, and make an additional assumption that  $G_{\rm m2}$  is negligible compared to  $\omega C_{\rm m2}$ . Then we can safely assume that  $G_{\rm m2}=0$  and obtain from Eq. 35

$$Y_{2} = G_{p}j\omega C_{m2}/(G_{p} + j\omega C_{m2})$$

$$= G_{p}j\omega C_{m2}(G_{p} - j\omega C_{m2})/[G_{p}^{2} + (\omega C_{m2})^{2}]$$

$$= G_{p}(\omega C_{m2})^{2}/[G_{p}^{2} + (\omega C_{m2})^{2}] + jG_{p}^{2}\omega C_{m2}[G_{p}^{2} + (\omega C_{m2})^{2}].$$

$$(47)$$

$$+ (\omega C_{m2})^{2}].$$

If  $Y_2 = A_2 + jB_2$ , where  $A_2$  and  $B_2$  are known real and imaginary parts of the admittance of the circuitry containing  $G_p$  and  $C_{m2}$ ,

$$A_2 = G_{\rm p}(\omega C_{\rm m2})^2 / [G_{\rm p}^2 + (\omega C_{\rm m2})^2], \tag{48}$$

$$B_2 = G_{\rm p}^2 \omega C_{\rm m2} / [G_{\rm p}^2 + (\omega C_{\rm m2})^2], \tag{49}$$

then

$$A_2^2 + B_2^2 = G_p^2(\omega C_{m2})^2 / [G_p^2 + (\omega C_{m2})^2].$$
 (50)

Dividing Eq. 50 by Eqs. 48 and 49, we obtain

$$G_{\rm p} = (A_2^2 + B_2^2)/A_2,$$
 (51)

$$C_{\rm m2} = (A_2^2 + B_2^2)/\omega B_2. \tag{52}$$

The first approach to the reconstruction of  $Y_2$  allows calculations of  $A_2$  and  $B_2$  from PSD signals. Then we can employ Eqs. 51 and 52 to find the values of  $G_p$  and  $C_{m2}$ . (This approach can be applied only if a stimulus sine wave is superimposed on a holding potential, and  $E_r$  is known and constant.)

If the PSD phase has been adjusted using NM, the admittance increment  $(Y_{incr})_{out} = R + jI$  can be used instead of  $A_2$  and  $B_2$  in Eqs. 51 and 52. According to Eq. 39,

$$A_2 = R/|T(\omega)|^2 \tag{53}$$

and

$$B_2 = I/|T(\omega)|^2. \tag{54}$$

Substituting these expressions for  $A_2$  and  $B_2$  into Eqs. 51 and 52, we obtain

$$G_{\rm p} = (R^2 + I^2)/(R|T(\omega)|^2),$$
 (55)

$$C_{m2} = (R^2 + I^2)/(\omega I |T(\omega)|^2),$$
 (56)

where  $|T(\omega)|^2$  is calculated according to Eq. F3 or F4 (footnote 4). Thus, having one sine wave in the stimulus signal

and employing the signals from both channels of a PSD, we can find  $G_p$  and  $C_{m2}$ . If we substitute the expression for  $Y_2$  from Eq. 47 into Eq. 37, then for  $Y_{incr}$  we will come to the formula identical to equation 21 of Lindau (1991). Our Eq. 56 in the particular case  $|T(\omega)|^2 = 1$  has also been derived by Lindau (1991, equation 23).

In the previous two sections we considered two approaches for finding parameters of the second (fusing) membrane. The modified MSW approach is more general, because it does not assume that  $G_{\rm m2}$  is negligible with respect to  $\omega C_{\rm m2}$ . This might be the case when low-frequency measurements are required or when the second membrane is excitable or leaky because of viral infection. However, the single sine wave approach is sometimes more practical, because it does not require a second sine wave and it can be used with an analog lock-in amplifier. The modified MSW approach requires more parameters and operations and thus can be disadvantageous when significant measurement errors occur.

Any implementation of these algorithms should take into account the effect of noise on nonlinear calculations based upon PSD measurements, as described in Zimmerberg (1993) for single-channel fusion pore calculations. One must exclude from the determination of the parameters of the second membrane not only the extremes for which the calculation formula are inapplicable, but also parts of an experiment in which the data lie inside so-called noise bounds around these extreme points, where the parameters of the second membrane should be considered undefined. (This approach has been implemented in the software Browse (see Materials and Methods). For instance, while calculating  $C_{\rm m2}$  according to Eq. 56, the software considers  $C_{\rm m2}$  undefined not only when I=0, but also in situations when I is small enough (does not exceed some multiple of the standard deviation of the imaginary channel calculated for the base line interval).) In such situations, having many different calculation procedures becomes especially valuable, because different algorithms generally have different "informationally undefined" areas, and an experimenter might be able to apply one algorithm to some intervals of data for which other algorithms do not work. For instance, if values  $g_1$  and  $g_2$  in Eq. 42 were close to each other, MSW would not work, because this equation contains their difference as a divider. On the other hand, application of the single sine wave method could still be possible in this case.

# Pore conductance can be calculated even if $G_{m1}$ changes in the course of an experiment

There are situations in which  $G_{\rm m1}$  is not constant during fusion. In the above calculations of  $Y_{\rm incr}$ , we denote the new conductance of the patch-clamped membrane when  $G_{\rm p}>0$  as  $G1_{\rm m1}=G_{\rm m1}+\Delta G_{\rm m1}$ . In this case, the admittance  $Y_{\rm m1+2}$  of the parallel combination of the first and the second membranes can be expressed as

$$Y_{\text{m1+2}} = G_{\text{m1}} + \Delta G_{\text{m1}} + j\omega C_{\text{m1}} + Y_2.$$

Substituting this expression into Eq. 33 and denoting  $Y_2 + \Delta G_{m1}$  as  $Y_G$ , we obtain

$$\begin{split} Y_{\rm incr} &= G_{\rm a} (G_{\rm m1} + j\omega C_{\rm m1} + Y_{\rm G}) / (G_{\rm a} + G_{\rm m1} + j\omega C_{\rm m1} + Y_{\rm G}) \\ &- G_{\rm a} (G_{\rm m1} + j\omega C_{\rm m1}) / (G_{\rm a} + G_{\rm m1} + j\omega C_{\rm m1}). \end{split}$$

This expression reduces to an equation that looks identical to Eq. 34 with respect to  $Y_G$ :

$$Y_{\text{incr}} = T(\omega)Y_{\text{G}}/(1/T(\omega) + Y_{\text{G}}/G_{\text{a}}). \tag{57}$$

If one of the two conditions below is true, we can neglect the ratio  $Y_G/G_a$ :

$$G_{\rm p} \ll G_{\rm a}$$
 and  $\Delta G_{\rm ml} \ll G_{\rm a}$ ,

or

$$G_{\rm m2} \ll G_{\rm a}$$
,  $\omega C_{\rm m2} \ll G_{\rm a}$ , and  $\Delta G_{\rm m1} \ll G_{\rm a}$ .

Then

$$Y_{\text{incr}} = T(\omega)^2 Y_{\text{G}}.$$
 (58)

Let us restrict ourselves to the second approach of  $Y_2$  restoration described above. We can then derive an equation similar to Eq. 39:

$$Y_{\rm G} = (Y_{\rm incr})_{\rm out}/|T(\omega)|^2. \tag{59}$$

Denoting  $(Y_{incr})_{out} = R + jI$ , we have

$$Y_2 = (R + jI)/|T(\omega)|^2 - \Delta G_{\text{ml}}.$$
 (60)

If  $G_{\rm m2}$  is negligible and  $E_{\rm r}$  is known and constant, exploiting the value  $G_{\rm DC}=I_{\rm h}/U_{\rm h}$  we can still use Eq. 18, even when the left part of the general circuit is present ( $G_{\rm p}>0$ ). From Eq. 18 we have

$$G_{\rm DC}(1/G_{\rm a} + 1/G_{\rm ml}) = (U_{\rm h} - E_{\rm r})/U_{\rm h}.$$
 (61)

Because we have assumed that  $G_{\rm a} \gg G_{\rm m1}$ , from Eq. 61 we can conclude that

$$Gl_{\rm m1} = G_{\rm DC}U_{\rm h}/(U_{\rm h} - E_{\rm r}).$$
 (62)

Using Eq. 62, we can calculate the change  $\Delta G_{\rm m1}$  in the  $Gl_{\rm m1}$  value, then employ Eq. 60 and single or multiple sine wave algorithms for calculation of fusion pore  $G_{\rm p}$  and (optionally) the capacitance  $C_{\rm m2}$ . In the particular case, when  $G_{\rm a} \gg \omega C_{\rm m1}$ , we can assume  $|T(\omega)|^2 \cong 1$ . Then from Eq. 60 we have

$$Y_2 = (R - \Delta G_{\text{m1}}) + jI.$$

This last equation is similar to Eq. F5 from footnote 4. It shows that when  $G_{\rm m1}$  changes and  $|T(\omega)|^2 \cong 1$ , rather than the pure in-phase signal of the PSD, we can simply use its adjusted value,  $R - \Delta G_{\rm m1}$ .

### **Practical considerations**

In systems where membrane fusion during exocytosis can be recorded at the level of a single event, a noninstantaneous increase in capacitance reflects the gradual expansion of a fusion pore (Zimmerberg et al., 1987). However, if capacitance is calculated on the basis of signals from both output channels of PSD, its true steplike shape can be restored (Lindau, 1991). Here we show that when  $G_p$  is small, and the frequency of the stimulating sine wave is high, such restoration is extremely sensitive to any phase errors. (The effect of small phase errors on the estimation of single cell parameters under the condition of a large decaying membrane conductance was recently investigated by Barnett and Misler (1997).) Let us assume that we have the following values of the elements of our general circuit: f = 2 kHz,  $R_{\rm a} = 4.8 \ {\rm M}\Omega, \, G_{\rm m1} = G_{\rm m2} = 0, \, C_{\rm m1} = 16.6 \ {\rm pF}, \, {\rm and} \, \, C_{\rm m2} =$ 1 pF. These values are characteristic of studies on virusinduced cell-cell fusion, which we discuss below. Suppose that the PSD signals reflect the admittance  $Y_2 = A_2 + jB_2$ , shifted by an angle  $\varphi_{err}$ :

$$(Y_2)_{\text{out}} = Y_2 \exp(j\varphi_{\text{err}}) = A_{\text{err}} + jB_{\text{err}}.$$

Substituting  $A_{\text{err}}$  and  $B_{\text{err}}$  into Eqs. 51 and 52, we obtain

$$C_{\rm err} = (A_{\rm err}^2 + B_{\rm err}^2)/\omega B_{\rm err} \tag{63}$$

and

$$G_{\rm err} = (A_{\rm err}^2 + B_{\rm err}^2)/A_{\rm err}, \tag{64}$$

where  $C_{\rm err}$  and  $G_{\rm err}$  are values of  $C_{\rm m2}$  and  $G_{\rm p}$ , distorted because of  $\varphi_{\rm err}$ . If we assume that  $G_{\rm p}$  linearly increases from 1 to 100 nS, we can analyze how calculations of these two parameters are affected by a phase offset. Fig. 2 A demonstrates that when  $G_{\rm p}$  is small, even a 2°-3° error in the phase settings can introduce substantial distortions of  $C_{\rm m2}$ . Such errors could be expected in practice, simply because the phase is always adjusted before fusion occurs. Fortunately, while the pore widens, the  $C_{\rm m2}$  computation error vanishes. On the contrary,  $G_{\rm err}$  deviates from  $G_{\rm p}$  only after substantial pore widening, indicating that we can safely measure small pores when the phase is not perfect (Fig. 2 B).

From Eq. 48 one can conclude that when the pore is narrow and the phase is set just about right,  $B_2$  is very small. Under these conditions, any deviations in phase introduce substantial errors in  $C_{\rm m2}$  because  $B_{\rm err}$  is a part of the denominator of Eq. 63. When the pore is large,  $A_2$  turns small, and the calculation of  $G_{\rm p}$  becomes sensitive to phase errors because of  $A_{\rm err}$  in the denominator of Eq. 64.

The phase sensitivity of the algorithm for the calculation of  $C_{\rm m2}$  explains some phenomena observed in practice. Experiments, shown in Fig. 3, were performed to study fusion between GP64-bearing insect cells and erythrocytes. Automated NM was used for phase adjustments;  $G_{\rm p}$  and  $C_{\rm m2}$  were calculated on the basis of two channels of the phase-sensitive software according to Eqs. 55 and 56. Red blood cells have a capacitance of  $\sim 1$  pF. When the phase was set correctly, the  $C_{\rm m2}$  trace rose almost instantaneously, i.e., it had a steplike shape (Fig. 3 A). However, usually the  $C_{\rm m2}$  trace was distorted because of a phase offset.

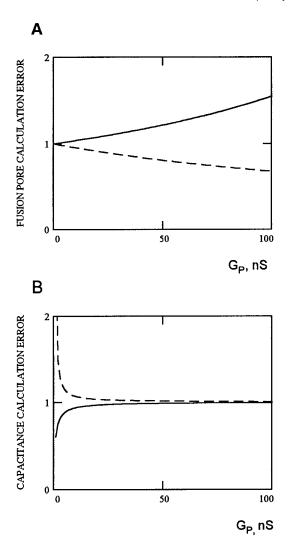
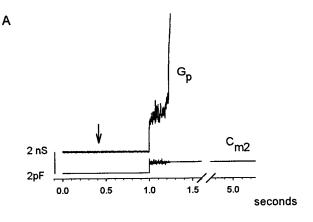


FIGURE 2 Computer simulations of the calculation errors induced by incorrect phase offsets. Calculations of  $C_{\rm m2}$  (A) and  $G_{\rm p}$  (B) were performed assuming that  $(Y_2)_{\rm out}$  (the measured value of the fusing circuitry admittance) deviates from  $Y_2$  because of a phase offset ( $\varphi_{\rm err}$ ). The values of the real and the imaginary components of ( $Y_2$ )<sub>out</sub> were computed, and distorted values of  $G_{\rm p}$ ,  $C_{\rm m2}$  ( $G_{\rm err}$ ,  $C_{\rm err}$ , respectively) were recalculated according to Eqs. 63 and 64. Phase-related errors were found by dividing  $C_{\rm err}$  by  $C_{\rm m2}$  and  $G_{\rm err}$  by  $G_{\rm p}$ ;  $\varphi_{\rm err}$  was equal to 3° (——) or to -3° (—). The parameters of the model circuit corresponded to those characteristic for Sf9 cells and erythrocytes.  $G_{\rm p}$  was increased linearly from 1 to 100 nS.

A more confusing situation arises when the phase is "undercompensated" ( $\varphi+2\phi<0$ ; see Eq. 24). In Fig. 3 B, the  $C_{\rm m2}$  trace lags fusion simply because the fusion-induced increment of the out-of-phase signal is less than the software detection limit (defined as the point that exceeds twice the standard deviation of the preceding base line). As soon as the out-of-phase signal was detected, the calculated value of  $C_{\rm m2}$  exceeded 1 pF. As predicted, this error vanished with the widening of the fusion pore. Adding +5° to the known experimental phase (in the off-line analysis of the data) recovered much of the natural shape of this capacitance increment, and the lag between the  $G_{\rm p}$  and  $C_{\rm m2}$  traces vanished. The adjustment of the shape of the  $C_{\rm m2}$  trace can



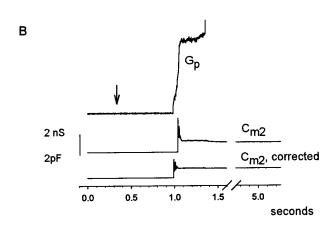


FIGURE 3 Off-line phase correction. Experiments on insect cell-erythrocyte fusion are used to demonstrate phase-related errors in the computation of membrane parameters. Here  $G_{\rm p}$  and  $C_{\rm m2}$  are calculated on the basis of the in-phase and out-of-phase software signals as described in Materials and Methods. (A) Fusion induces a steplike increment of the  $C_{\rm m2}$  trace, indicating that in this experiment, phase was adjusted correctly. Background noise in the  $G_{\rm p}$  trace is simply that of the in-phase software channel; before fusion occurred,  $C_{\rm m2}$  was defined as zero. Here and in B, the arrow shows the onset of a pressure pulse delivering acid solution to the cell surface. (B) The characteristic overshoot in  $C_{\rm m2}$  (middle trace) indicates that at the moment when fusion has occurred, the phase deviated from its desired value. Off-line addition of 5° to the experimental phase value improves the shape of  $C_{\rm m2}$  (bottom trace), but does not affect  $G_{\rm p}$  (not shown). The adjustment of the  $C_{\rm m2}$  shape can be used to adjust the phase off-line.

be used for the final correction of the phase setting long after an experiment has been finished.

#### **DISCUSSION**

Studies on membrane fusion are conducted using a wide variety of modern methods, including biochemistry, molecular biology, and x-ray crystallography. Compared to other functional assays, electrophysiological methods provide faster time resolution (up to hundreds of microseconds) and higher amplitude resolution (the fusion of a single submicron intracellular vesicle with the plasma membrane (Lollike et al., 1995). Changes in capacitance monitor fusion: it is the conductance and kinetics of formation and widening

of the fusion pore that contain information about its immediate mechanisms.

The calculation of membrane parameters starts with an accurate measurement of the source (cell and pipette) admittance. The source admittance, before entering a PSD, is distorted by low-pass filtering due to different pieces of electrophysiological equipment. Some of the approaches of the Lindau-Neher type are sensitive to this phase offsetinduced distortion (i.e., they do not have built-in phase adjustment procedures). A few methods of resolving this problem have been proposed. Analog phase shifters are used by Rohlicek and Schmid (1994) to null out the phase offset. Gillis (1995) describes an empirical procedure, suggesting the insertion of a capacitor or a resistor into the input of an amplifier and a subsequent compensatory tuning of the lock-in phase angle. The phase found by this technique is affected by the signal frequency, analog filter settings, and sampling rate, and therefore it should be redefined quite often. Donnelly (1994) suggested the following procedure: 1) an outside source generates a signal with known amplitude and phase with respect to a stimulus sine wave; 2) this signal is applied to the equipment (filter) and 3) is acquired for subsequent calculation of the filter compensation vector, which 4) is subsequently used to correct experimental signals. We propose the use of the patch-clamp amplifier compensation circuitry as a source for such a signal. A displacement of a C-slow potentiometer simulates changes in the source admittance. Such changes, recorded by the phase-sensitive software, are compared to its theoretically expected amplitude. The phase is then calculated based on the divergence of these two values, which is caused by the equipment-related phase offset. Our new procedure for determination of phase distortion can be built into LN or MSW. This method of cell-pipette admittance reconstruction does not require the additional hardware suggested by Donnelly (1994) or Rohlicek and Schmid (1994), nor does it need readjustments, characteristic of the empirical procedure, proposed by Gillis (1995).

The LN approach imposes strict requirements on the membrane resting potential. For proper implementation, it requires  $E_{\rm r}$  to be equal to zero or to be known and constant, a condition that may not occur in practice, because of changes in the ionic composition of a cell during an experiment. MSW solves this problem: one can calculate  $R_{\rm a}$ ,  $C_{\rm m1}$ , and  $G_{\rm m1}$ , ignoring  $E_{\rm r}$ . However, if stimulating sine waves are superimposed on the holding potential,  $E_{\rm r}$  can easily be calculated, which may aid in the simultaneous recording of changes in cell conductance and capacitance.

Unlike LN, PL has a built-in phase adjustment procedure that requires dithering of two potentiometers: C-slow and the one in the PSD phase circuitry. Our own experience shows that a graceful performance of this procedure requires some practice. Fortunately, the NM technique of the phase adjustment can easily be automated. A displacement of a C-slow potentiometer can be used to calculate the argument of the corresponding signal. If the argument deviates from  $3\pi/2$ , the software phase is adjusted to correct

this deviation. Thus calibration (finding  $|T(\omega)|^2$ ) and phase setting can be combined in a software implementation of PL.

Compared to PL, the advantage of MSW is the calculation of all three parameters of the first membrane. Although MSW in its initial form does not consider the fusion pore, we show how knowledge of  $R_a$ ,  $C_{m1}$ ,  $G_{m1}$  and the phase can be used to restore values of all three elements of the second (fusing) membrane. Indeed, if all elements of the first cell are known, the admittance of the fusing circuitry  $Y_2$  can be restored, and subsequent calculation of  $G_p$ ,  $G_{m2}$ , and  $C_{m2}$ can be performed under conditions that are not as strict as the ones imposed by other PL-based methods (Breckenridge and Almers, 1987; Lindau, 1991). Our modification of MSW requires only the constancy of  $R_a$ ,  $C_{\rm m1}$  during fusion. Any PL-based method suggests an accurate and long-lasting compensation of first cell parameters (i.e.,  $G_{\rm sr} \cong 1/R_{\rm a}$  and  $C_{\rm sl} \cong C_{\rm m1}$ ). Furthermore,  $G_{\rm m2}$ , the element neglected by other methods, can be calculated by our algorithm.

Finally, if all attempts to find the correct phase have failed and  $\varphi$  at the very moment of fusion deviated from the desirable angle, this deviation can be detected off-line by examination of the capacitance trace shape. Theoretically, the fusion-induced increment of the capacitance trace should look like a step function. However, if  $C_{\rm m2}$  is calculated using only the out-of-phase PSD signal, the  $C_{\rm m2}$  trace rises gradually, indicating the development of a fusion pore. Obviously, a command potential drops across  $G_{\rm p}$ ,  $C_{\rm m2}$  in series in such a way that if the pore is narrow, a resulting current partially escapes the out-of-phase signal and shows up in the in-phase signal. Lindau (1991) introduced an algorithm of capacitance shape restoration that uses both PSD signals for the calculation of  $C_{\rm m2}$ . If this algorithm has been applied, and  $C_{\rm m2}$  still does not look "perfect," a deviation of the phase from its desired value can be suspected. Off-line adjustment of the phase can restore the steplike increment of  $C_{\rm m2}$  required by membrane fusion. The opposite is also true: such restoration can be a good indicator of a successful phase angle correction. The off-line phase setting procedure suggested here has more than just a cosmetic value. Although pore conductance calculations show a relative insensitivity to the phase offset when  $G_p$  is small, this error increases with pore widening. Thus off-line phase tuning can be included in the methodological repertoire of those researchers who are interested in exploring the dvnamics of pore widening and the forces governing pore enlargement.

The modified MSW and the off-line phase setting presented here could be of special importance for those studying virus-induced cell-cell fusion, where pores can be "frozen" in their low conductive states for a long period of time, and the specific resistance of the second membrane could be higher than that of a small exocytotic granule.

Chernomordik and E. Leikina for sharing their findings regarding GP64-induced Sf9/RBC fusion; E. Leikina for preparation of erythrocytes; and O. Plonskaia for help with the manuscript.

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